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Vector Mesons in Nuclear Medium – an Effective Lagrangian Approach –

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Abstract

Effective masses of ρ and ω mesons in nuclear medium are studied in a hadronic effective theory. Both the pole position and the screening mass decrease in nuclear matter due to the polarization of the nucleon Dirac sea. The physical origin of the decrease is a reduction of the wave function renormalization constant induced by the tensor (vector) interaction of the ρ (ω) with the nucleon. Relation to the results of the QCD sum rules is also discussed.

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1. INTRODUCTION. Effective masses of the vector mesons (ρ, ω, ϕ) in nuclear medium [1, 2] have recently attracted wide interests. The decrease of the masses in nuclei is interpreted as an evidence of the partial restoration of chiral symmetry [3]. Direct experiments using the e^+e^- production in $\gamma - A$ and $p - A$ processes are also proposed to check the mass shift [4].[3]

As has been shown by Lee and one of the present authors using the QCD sum rules [1], the ρ and ω masses around the nuclear matter density ρ_0 can be parametrized as

$$m^*_{\rho,\omega}/m_{\rho,\omega} \simeq 1 - (0.18 \pm 0.05)(\rho/\rho_0), \tag{1}$$

with $m^*_{\rho,\omega}$ ($m_{\rho,\omega}$) being the pole position of the ρ and ω propagators in the medium (vacuum). Brown and Rho also predicted similar decrease on the basis of the dilatational symmetry in the chiral lagrangian [2]. In these approaches, the decrease is intimately related to the chiral structure of the QCD vacuum in the presence of matter. On the other hand, in the conventional hadronic approaches where only the polarization of the nucleon Fermi sea is taken into account, the masses of the vector mesons stay constant or increase only slightly [6, 7].

The purpose of this article is twofold: First one is to show explicitly that $m^*_{\rho,\omega}$ do decrease even in the level of the hadronic effective theory if one properly takes into account the fluctuation of the nucleon Dirac sea. We will also analyze the physical origin of the decrease. Another purpose is to clarify the difference between the "real mass" defined by the pole position of the propagator and the "screening mass" defined by the damping in the space like region of the propagator. They are equal in the vacuum but different in the medium, and the distinction between the two should be made carefully when one analyses experiments with different kinematics. So far, the effect of the Dirac sea has been studied only for the real mass of the ω-meson [8, 9, 10, 11]. As for ρ, it is not obvious whether the real and screening masses decrease with the same manner as m^*_ω in hadronic models: In fact, the ωNN interaction is dominated by the vector coupling, while the ρNN interaction is dominated by the tensor coupling as is known from nucleon form factors and the nuclear forces [12]. We will show that the ρNN tensor coupling actually plays a crucial role for m^*_ρ.

2. LAGRANGIAN. Let's start with an interaction lagrangian of ρ, ω with the nucleon:

$$L_{int} = g_\alpha \left[\overline{\psi}\gamma_\mu \tau^a \psi - \frac{\kappa_\alpha}{2M} \overline{\psi}\sigma_{\mu\nu}\tau^a\psi\partial^\nu \right] V_a^\mu, \quad \alpha = \{\rho, \omega\} \ , \tag{2}$$

[3]See also [5] for the vector mesons in hot matter as a signal of the formation of the quark gluon plasma in the relativistic heavy ion collisions.

where a runs from 0 through 3, V_0 (V_{1-3}) corresponds to the ω (ρ) field, τ^a is the isospin matrix with τ^0=1, and M is the nucleon mass. The numerical values of the coupling constants (g_α, κ_α) will be given in section 4.

In the one-loop level, the density dependent part of the self-energy comes only from the nucleon-loop[4]:

$$\Pi^{ab}_{\mu\nu}(q) = -\frac{i}{(2\pi)^4}\int d^4k \mathrm{Tr}[\Gamma^a_\mu G(k+q)\Gamma^b_\nu G(k)] \quad , \tag{3}$$

where (a,b) are the isospin indices and

$$\Gamma^a_\mu = g_\alpha[\gamma_\mu\tau^a - \frac{\kappa_\alpha}{2M}\sigma_{\mu\lambda}iq^\lambda\tau^a], \qquad \Gamma^b_\nu = g_\alpha[\gamma_\nu\tau^b + \frac{\kappa_\alpha}{2M}\sigma_{\nu\lambda}iq^\lambda\tau^b] \quad . \tag{4}$$

The nucleon propagator in the medium $G(k) = G_F + G_D$ reads

$$G_F = \frac{\gamma\cdot k^* + M^*}{k^{*2} - M^{*2} + i\epsilon}, \tag{5}$$

$$G_D = i\frac{\pi}{E(k)}(\gamma\cdot k^* + M^*)\delta(k^*_0 - E(k))\theta(k_F - |\mathbf{k}|) \quad . \tag{6}$$

Here $k^{*\mu} \equiv (k^0 - g_\omega\langle V^0\rangle, \mathbf{k})$ with $\langle\cdot\rangle$ being the ground state expectation value, $E(k) = \sqrt{\mathbf{k}^2 + M^{*2}}$ and k_F is the fermi momentum. As for the effective nucleon mass M^*, we adopt the result of the relativistic Hartree approximation with vacuum fluctuation [6] which is shown in Fig. 1. $\Pi_{\mu\nu}$ in (3) is composed of two parts $\Pi_{\mu\nu} = \Pi^{0F}_{\mu\nu} + \Pi^D_{\mu\nu}$: the first term corresponds to the fluctuation of the Dirac sea of the nucleons with mass M^*, while the second term corresponds to the fluctuation of the Fermi sea and the Pauli blocking. $\Pi^{0F}_{\mu\nu}$ generally has divergences to be subtracted. We will show our subtraction procedure in section 3 and define $\Pi^F_{\mu\nu}$ as the subtracted polarization.

The vector meson propagator in the medium has a general form

$$D_{\mu\nu} = \frac{-P_L}{q^2 - m^2 + \Pi_L} + \frac{-P_T}{q^2 - m^2 + \Pi_T}, \tag{7}$$

where we have suppressed isospin indices (a,b). m denotes the ρ or ω mass in the vacuum and P_T (P_L) is the projection operator [5] to the transverse (longitudinal) direction to $\mathbf{k}$. $\Pi_{T,L}$ is related to $\Pi_{\mu\nu}$ as $\Pi_L = -(q^2/\mathbf{q}^2)\Pi_{00}, \Pi_T = (\Pi^l_l + (q_0^2/\mathbf{q}^2)\Pi_{00})/2$. To obtain (7), we have used the Steukelberg propagator with $\lambda \to \infty$ [13] as a free propagator of the massive vector mesons.

[4]The self interaction of the ρ meson gives density dependence only from two or higher loops. The coupling of ρ with in-medium pions analyzed in [7] is also the higher loop effect and will not be considered in this paper.

[5]$P^{\mu\nu}_T = g^{\mu i}(g_{ij} + k_i k_j/\mathbf{k}^2)g^{\nu j}$ and $P^{\mu\nu}_L = e^\mu e^\nu$ with $e^\mu = \frac{i}{\sqrt{k^2}}(|\mathbf{k}|, \frac{k_0}{|\mathbf{k}|}\mathbf{k})$.

3. SUBTRACTION PROCEDURE. The interaction (2) is not renormalizable in the conventional sense. This does not, however, cause essential difficulties, since the requirement of the strict renormalizability is not necessary in effective theories. A typical example is the non-linear σ model as a low energy effective theory of QCD. The model contains infinite series of the higher dimensional operators which play a role to cancel the divergences emerging from the loops of the lower dimensional operators [14]. In this letter, instead of developing a systematic subtraction procedure, we will take a phenomenological way to extract $\Pi^F_{\mu\nu}$ from $\Pi^{0F}_{\mu\nu}$. First of all, we are interested only in the density dependence of $m^*_{\rho,\omega}$, thus we will subtract away both divergent and finite parts coming from the nucleon-loop at zero density. This corresponds to a set of the renormalization conditions $\partial^n \Pi^F(q^2)/\partial(q^2)^n \mid_{q^2=m^2} = 0$ $(n = 0, 1, 2, \cdots \infty)$ or equivalently the infinite series of counter terms which normalize the propagator in the vacuum to $1/(q^2 - m^2)$. At finite density, we will adopt similar conditions $\partial^n \Pi^F(q^2)/\partial(q^2)^n \mid_{M^* \to M, q^2=m^2} = 0$ $(n = 0, 1, 2, \cdots \infty)$. This together with a condition that the higher dimensional counter terms contain only polynomials with respect to the hadron fields, one can uniquely single out the density dependent part from $\Pi^{0F}_{\mu\nu}$. Although our procedure is physically plausible and does not suffer from the Landau ghost problem [9], it is still "a" way to subtract the divergences among many other possibilities. For the ω meson, our procedure is equivalent to that adopted in [10]. We have also checked that the qualitative results presented in this paper are not affected even when we take other subtraction schemes given in [8, 9, 11].

Using the dimensional reguralization and the above subtraction procedure, one obtains the following $\Pi_{\mu\nu}$ for the ρ-meson. ($Q_{\mu\nu} \equiv q_\mu q_\nu / q^2 - g_{\mu\nu}$).

$$\Pi^{ab}_{\mu\nu} = \delta^{ab} \left(Q_{\mu\nu} \Pi^F + \Pi^D_{\mu\nu} \right), \tag{8}$$

$$\Pi^F = \Pi^F_v + \Pi^F_{v,t} + \Pi^F_t, \quad \Pi^D_{\mu\nu} = \left(\Pi^D_v + \Pi^D_{v,t} + \Pi^D_t \right)_{\mu\nu}, \tag{9}$$

$$\Pi^F_v = \frac{g_\rho^2}{\pi^2} q^2 \int_0^1 dx\ x(1-x) \log \left\{ \frac{M^{*2} - q^2 x(1-x)}{M^2 - q^2 x(1-x)} \right\}, \tag{10}$$

$$\Pi^F_{v,t} = \left(\frac{g_\rho^2 \kappa_\rho}{2M}\right) \frac{M^* q^2}{\pi^2} \int_0^1 dx \log \left\{ \frac{M^{*2} - q^2 x(1-x)}{M^2 - q^2 x(1-x)} \right\}, \tag{11}$$

$$\Pi^F_t = \left(\frac{g_\rho \kappa_\rho}{2M}\right)^2 \frac{q^2}{2\pi^2} \int_0^1 dx \{M^{*2} + q^2 x(1-x)\} \log \left\{ \frac{M^{*2} - q^2 x(1-x)}{M^2 - q^2 x(1-x)} \right\}, \tag{12}$$

$$(\Pi^D_v)_{\mu\nu} = g_\rho^2 \frac{\Pi^\omega_{\mu\nu}(q)}{g_\omega^2}, \tag{13}$$

$$(\Pi^D_{v,t})_{\mu\nu} = Q_{\mu\nu} \left(\frac{g_\rho^2 \kappa_\rho}{2M} \right) 4 M^* q^2 I_0(q), \tag{14}$$

$$(\Pi^D_t)_{\mu\nu} = \left(\frac{g_\rho \kappa_\rho}{2M} \right)^2 q^2 \left[\frac{-\Pi^\omega_{\mu\nu}(q)}{g_\omega^2} + Q_{\mu\nu} \left\{ (4M^{*2} + q^2) I_0(q) + \frac{\rho_\sigma}{M^*} \right\} \right]. \tag{15}$$

Since $\Pi^{\omega}_{\mu\nu}(q)$, $I_0(q)$ and ρ_σ are defined in ref.[8], we will not recapitulate them here. $\Pi_{\mu\nu}$ for the ω meson is simply obtained by the replacement $(g_\rho, \kappa_\rho) \rightarrow (g_\omega, \kappa_\omega)$ in the above formulas. We have neglected the mixing of ω with the scalar meson σ which does not modify our results qualitatively.[6]

4. COUPLING CONSTANTS. We will take the two sets of the coupling constants given in Table 1. $\kappa_\omega = 0$ is taken in both sets, since the ωNN tensor coupling is generally small (e.g. $\kappa_\omega = 0.12$ in the vector dominance model).

	set I	set II
g_ρ	2.63	2.72
κ_ρ	6.0	3.7
g_ω	10.1	10.1

Table 1: Two different sets of the coupling constants adopted in Fig. 2 and Fig. 3.

Set I is obtained from the $N - N$ forward dispersion relation [16]. The Bonn potential of the $N - N$ force gives similar values with this set. Set II is obtained by the vector-meson dominance together with the ρ universality [17]. In the latter case, one cannot determine the ωNN coupling, therefore we adopted the same coupling with the one in the first set. A major difference between the two sets is the strength of the ρNN tensor coupling. (See ref.[12] for the detailed discussion on the vector-meson coupling constants.) In our calculations, vertex form factors are not taken into account for simplicity.

5. REAL, SCREENING and INVARIANT MASSES. In the following, we will focus on the transverse polarization Π_T defined in eq. (7) and consider the inverse propagator

$$D_T^{-1}(q_0, |\mathbf{q}|) = q^2 - m^2 + \Pi_T^D(q_0, |\mathbf{q}|) + \Pi_T^F(q^2) \quad . \tag{16}$$

Detailed account including the discussion on Π_L will be given elsewhere [18].

Let us define three kinds of masses m^*_{re} (real mass), m^*_{inv} (invariant mass) and m^*_{sc} (screening mass). m^*_{re} is defined as a lowest zero of $D_T^{-1}(q_0, 0)$. It is the quantity to be compared with that in the QCD sum rules (eq.(1)) and is related to the peak position of the e^+e^- spectrum obtained from the decays of vector mesons

[6]Our $\Pi^{F,D}$ for the ω meson agree with the previous calculations [8, 9, 10, 11] except for the different subtraction procedure. Π^D for the ρ meson agrees with ref.[15] except that $M^* = M$ is taken and the sign of $\Pi^D_{v,t}$ is opposite to ours in [15]. Our formulas of Π^F for the ρ meson are new.

in nuclei [4]. The invariant mass m^*_{inv} is defined as a lowest zero of D_T^{-1} with Π_T^D neglected, in which case D_T^{-1} is a function of q^2 only. m^*_{inv} here contains only the fluctuation of the Dirac sea by definition. Since the rotational invariance leads to the equality $\Pi_T^D(q_0,0) = \Pi_L^D(q_0,0)$, m^*_{re} and m^*_{inv} turn out to be the common poles in both longitudinal and transverse part of the vector meson propagator. Finally, we define the screening mass m^*_{sc} as a pure imaginary zero of $D_T^{-1}(0,|\mathbf{q}|)$. If there is such a pole at $|\mathbf{q}| = im^*_{sc}$, it contributes to the meson propagator in the coordinate space as $D_T(t=0,\mathbf{x}\to\infty) \sim \exp(-m^*_{sc}|\mathbf{x}|)$.[7] In general, m^*_{sc} for the transverse propagator takes different value from that in the longitudinal one.

If we neglect the Fermi sea polarization Π^D, one has a simple relation between the invariant mass m^*_{inv} and the finite wave-function renormalization Z in medium. Since $\Pi_T^F(=\Pi_L^F)$ is proportional to q^2 as can be seen from eq.(9)-(12), the meson propagator near the mass shell is written as

$$D_{\mu\nu} \sim \frac{1}{Z^{-1}q^2 - m^2} = \frac{Z}{q^2 - Zm^2} \quad , \tag{17}$$

which leads to $m^*_{inv} = \sqrt{Z}m$. Z is nothing but the probability to find physical $\rho(\omega)$ in the vacuum inside the physical $\rho(\omega)$ in the medium. Note that Z can be larger or smaller than unity depending on the sign of $M^* - M$.

6. NUMERICAL RESULTS AND DISCUSSIONS. In Fig. 1, the effective masses of ω are shown together with M^*/M. The dashed line denotes m^*_{inv}/m. One sees that m^*_{re}, m^*_{inv} and m^*_{sc} all decrease at finite density, e.g. $m^*_{re}/m \simeq 0.8$ at $\rho = \rho_0$. By comparing m^*_{re} with m^*_{inv}, one also observes that there are two competing effects: (a) polarization of the Dirac sea of the nucleons with M^* which tends to decrease m^*_{inv}, and (b) polarization of the Fermi sea and the Pauli blocking which contributes positively to m^*_{re}. We found that (a) dominates over (b). Our result for m^*_{re} is also consistent with that in the previous analyses [8, 9, 10, 11].

There is a physical reason why m^*_{inv} decreases in the medium: Since $M^* < M$ in nuclear matter, it is easier for vector mesons to dissociate into the $N\bar{N}$ pair in the medium than in the vacuum. In other words, physical ω is more dressed by the $N\bar{N}$ pairs in the medium, which leads to $Z < 1$ and $m^*_{inv}/m = \sqrt{Z} < 1$.

In Fig. 2 (Fig. 3), we have shown the effective masses of the ρ meson with the parameter set I (II). The strong ρNN tensor coupling plays a dominant role and gives $m^*_{re}/m \simeq 0.6 - 0.7$ at $\rho = \rho_0$. The polarization of the Dirac sea is again the most important ingredient and the suppression of Z is the main reason for the mass

[7]Strictly speaking, $D_T(0,|\mathbf{q}|)$ has also cuts in the complex $|\mathbf{q}|$ plane which give rise to the Friedel oscillations in $D_T(t=0,\mathbf{x}\to\infty)$. We will not consider such contribution in this paper.

reduction. Note also that $m^*_{sc} < m^*_{inv}$ for the ρ-meson, which is opposite to the ω-meson case.

It is in order here to make final remarks:
(i) The reduction of m^*_{re}/m is consistent with that in the QCD sum rules (eq.(1)) for the ω meson, and even larger reduction is observed for the ρ-meson in this paper. One should, however, remember that we did not consider any vertex form factors for the ρNN and ωNN couplings. Such form factors will attenuate the magnitude of the mass shift of the ρ-meson and the ω-meson in a different way. From Fig. 1-3, one also observes considerable non-linearity of m^*_{re} as a function of density. This is contrast to the linear dependence in eq.(1). Further study is necessary to clarify the origin of this difference.
(ii) m^*_{re} has direct relevance to the production of lepton pairs as we have mentioned [4, 5]. On the other hand, m^*_{sc} is related to the t-channel exchange of the vector mesons in nuclear processes such as $K^+ - {}^{12}\mathrm{C}$ scattering [3]. In this case, however, one should also take into account the reduction of Z in (17) which partly cancels the effect of the mass reduction as shown in [10].
(iii) Despite some quantitative differences between the result of the effective theory here and that in the QCD sum rules, the physical origin of the decreasing m^*_{re} is quite similar in two approaches. The driving forces of the mass reduction are the fluctuation of the Dirac sea in the effective theory and the change of the chiral condensate $\langle(\bar{q}q)^2\rangle$ in the QCD sum rules. Physically they are both related to the structure of the QCD vacuum in nuclear matter. On the other hand, the fluctuation of the Fermi sea (particle-hole excitations) in the effective theory and the twist 2 condensate $\langle\bar{q}\gamma_\mu D_\nu q\rangle$ in the QCD sum rules contribute positively to m^*_{re}. They can be interpreted as the scattering of the vector mesons by the valence nucleons in the nuclear matter. One also finds that Dirac beats Fermi in both approaches.

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Figure Captions

Fig. 1: Effective masses of the nucleon and the ω meson as a function of the baryon density. m^*_{re}, m^*_{sc} and m denote the real mass, screening mass and the mass in the vacuum, respectively. The dashed line corresponds to the invariant mass in medium m^*_{inv}/m.

Fig. 2: Real, screening and invariant masses of the ρ-meson in the parameter set I. The dashed line corresponds to m^*_{inv}/m.

Fig. 3: Same quantities with Fig. 2 in the parameter set II.

References

[1] T. Hatsuda and Su H. Lee, Phys. Rev. **C46** (1992) R34.

[2] G. E. Brown and M. Rho, Phys. Rev. Lett. **66** (1991) 2720.

[3] G. E. Brown, C. J. Dover, P. B. Siegel and W. Weise, Phys. Rev. Lett. **60**, 2723 (1988). K. Kubodera and M. Rho, Phys. Rev. Lett. **67** (1991) 3479.

[4] H. Shimizu, private communication. H. Enyo, private communication.

[5] T. Hatsuda, Y. Koike and Su H. Lee, Nucl. Phys. **B394** (1993) 221; T. Hatsuda, *QCD and Quark/Hadron Matter*, preprint UTHEP-271 (1994).
K. Yagi, *PHENIX Experiment at BNL-RHIC*, Univ. of Tsukuba report (1994).

[6] S. A. Chin, Ann. Phys. **108** (1977) 301.

[7] M. Herrmann, B. L. Friman and W. Nörenberg, Nucl. Phys. **A560** (1993) 411.
M. Asakawa, C. M. Ko, P. Lévai and X. J. Qiu, Phys. Rev. **C46** (1992) R1159.

[8] H. Kurasawa and T. Suzuki, Nucl. Phys. **A490** (1988) 571; Prog. Theor. Phys. **84** (1990) 1030.

[9] K. Tanaka, W. Bentz and A. Arima, Nucl. Phys. **A528** (1991) 676.

[10] J. C. Caillon and J. Labarsouque, Phys. Lett. **B331** (1993) 19.

[11] H.-C. Jean, J. Piekarewicz and A. G. Williams, Florida State Univ. preprint FSU-SCRI-93-132 (1993).

[12] R. Machaleit, in *Advances in Nuclear Physics*, edited by J. W. Negele and E. Vogt (Plenum, New York, 1986), Vol. 19.

[13] C. Itykson and J-B. Zuber, *Quantum Field Theory*, section 12-5-2, (McGraw-Hill, New York, 1980).

[14] S. Weinberg, Physica **96A** (1979) 327. J. Gasser and H. Leutwyler, Ann. Phys. **158** (1984) 142.

[15] L. S. Celenza, A. Pantziris and C. M. Shakin, Phys. Rev. **C 45** (1992) 45.

[16] W. Grein, Nucl. Phys. **B131** (1977) 255; W. Grein and P. Kroll, Nucl. Phys. **A338** (1980) 332.

[17] J. J. Sakurai, *Currents and Mesons*, (Univ. of Chicago Press, Chicago, 1969).

[18] H. Shiomi and T. Hatsuda, in preparation.